

SEMINARIO
CONJUNTO MODELAMIENTO ESTOCÁSTICO Y NÚCLEO MESCD

EXPOSITOR
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(CMM Y UAI)

TÍTULO

LIMIT DISTRIBUTIONS RELATED TO THE EULER DISCRETIZATION ERROR OF BROWNIAN MOTION ABOUT RANDOM TIMES

Resumen:

In this talk we study the simulation of barrier-hitting events and extreme events of one-dimensional Brownian motion. We call "barrier-hitting event" an event where the Brownian motion hits for the first time a deterministic "barrier" function; and call "extreme event" an event where the Brownian motion attains a minimum on a given compact time interval or unbounded closed time interval. To sample these events we consider the Euler discretization approach of Brownian motion; that is, simulate the Brownian motion on a discrete and equidistant times mesh, e.g., $\{0, 1/n, 2/n, \dots\}$. With this, for each of the aforementioned events we study the discretization error between the actual time the event occurs versus the time the event occurs on the discretized path, and also the discretization error on the position of the Brownian motion at these times.

Our main results are threefold.

First, we show limits in distribution for the discretization errors normalized by their convergence rate, and give closed-form analytic expressions for the limiting random variables.

Second, we use the previous limits in distribution to derive new limits in the theory of Diffusion Approximation, which studies approximating random walks by using diffusion processes. More precisely, we obtain limits that use Brownian motion to approximate the asymptotic behavior of Gaussian random walks in the following situations: (1.) the overshoot of a Gaussian walk above a barrier that goes to infinity; (2.) the minimum of a Gaussian walk over a time horizon that goes to infinity; and (3.) the global minimum of a Gaussian walk having positive drift decreasing to zero.

Third, and perhaps more important, we provide a unified framework that relates several papers since the 1960's where the constant $-\zeta(1/2)/\sqrt{2\pi}$ has appeared, where ζ is the Riemann zeta function. Up to now, how these works were precisely connected has been considered an open question. We show that this constant is the mean of some of the limiting distributions we obtain, and claim that each of these papers is directly connected to the results we derive.

Fecha: Martes 28 de Marzo, 16:30 hrs

Lugar, Sala John Von Neumann, 7mo piso, Centro de Modelamiento Matemático.

